

# Free convection in porous media filled right-angle triangular enclosures <sup>☆</sup>

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Available online 27 September 2006

## Abstract

Steady-state free convection heat transfer in a right-angle triangular enclosure, whose vertical wall insulated and inclined and bottom walls are differentially heated, is performed in this study. The governing equations are obtained using Darcy model. In this study, the governing equations were solved by finite difference method and solution of algebraic equations was made via Successive Under Relaxation method. The effect of aspect ratios ranging from 0.25 to 1.0 and Rayleigh numbers  $50 \leq Ra \leq 1000$  is investigated as governing parameters on heat transfer and flow field. It is observed that heat transfer is increased with the decreasing of aspect ratio and multiple cells are formed at high Rayleigh numbers.

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*Keywords:* Porous medium; Natural convection; Triangular enclosure

## 1. Introduction

Analysis of natural convection heat transfer in triangular enclosures can be found in the design of building roofs and attics, solar energy collectors, cooling of electrical devices as PC, TV etc. It arises in these geometries due to temperature difference between inside heating and environmental conditions. Natural convection in triangular enclosures under different thermal boundary conditions in non-porous media using air as working fluid has been extensively studied in the past years [1–9].

Natural convection heat transfer in porous media have wide range of applications in engineering such as solar collectors, insulation for buildings, nuclear waste management, geothermal applications. Those applications were reviewed by Nield and Bejan [10] and Ingham and Pop [11]. The natural convection in enclosures can be classified in two main groups as rectangular enclosures [12–22] and non-rectangular enclosures [23–29]. However, the number of studies on natural convection in porous non-rectangular geometries is very limited. Kumar and Kumar [23] conducted a numerical study on natural convection in porous trapezoidal enclosures using finite element method. They indicated

<sup>☆</sup> Communicated by W.J. Minkowycz.

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## Nomenclature

AR	Aspect ratio, H/L
$g$	Gravitational acceleration
$Gr$	Grashof number
$K$	Permeability
$L$	Length of the bottom wall
$H$	Height of triangle
$Nu$	Nusselt number
$Pr$	Prandtl number
$Ra$	Rayleigh number
$u, v$	Axial and radial velocities
$x, y$	Non-dimensional coordinates

### Greek Letters

$\nu$	Kinematic viscosity
$\theta$	Non-dimensional temperature
$\beta$	Thermal expansion coefficient
$\alpha$	Thermal diffusivity
$\psi$	Non-dimensional stream function

### Subscript

c	cold
h	hot

that Nusselt number increases with the increasing angle of inclination of the walls of the trapezoidal enclosures. Murthy et al. [24] investigated the natural convection heat transfer from a horizontally wavy surface in a porous enclosure. Kumar [25] and Kumar and Shalini [26] investigated free convection in wavy vertical porous enclosures. Baytas and Pop [27] performed a numerical analysis to investigate the natural convection in a porous trapezoidal enclosure. They solved the Darcy and energy equations using Alternative Direction Implicit finite-difference method. They indicated that conduction heat transfer regime is dominated for low Rayleigh numbers. The same method was used to investigate free convection in oblique enclosures filled with a porous medium in their other study [28]. They observed that near the sharp corners of the oblique enclosure the flow and temperature break down into a series of subvortices. Das and Morsi [29] studied a non-Darcian numerical modeling in dome-shaped, heat-generating, porous enclosures. Their results show a significant change in temperature distribution in comparison to rectangular enclosures because of the existence of the dome-shaped top adiabatic cover.

The main purpose of this study is to investigate the free convection heat transfer in a right-angle triangular enclosure with heated from bottom wall and cooled inclined wall. The tests were performed for different aspect ratios of triangular enclosure and Rayleigh numbers. According to knowledge of the authors and the literature given above, there is no study on natural convection heat transfer in triangular enclosure. Thus, the study may be considered as a tool for scientific researches.

The schematic configuration, boundary conditions and coordinate system were depicted in Fig. 1 that shows two-dimensional triangular enclosure with the bottom wall length  $L$  and the height of the vertical wall  $H$ . The enclosure is filled with porous media.

## 2. Governing equations

To write the governing equation, some assumptions were made: The properties of the fluid and the porous media are constant; the cavity walls are impermeable; the Boussinesq approximation is valid; and the viscous drag and inertia

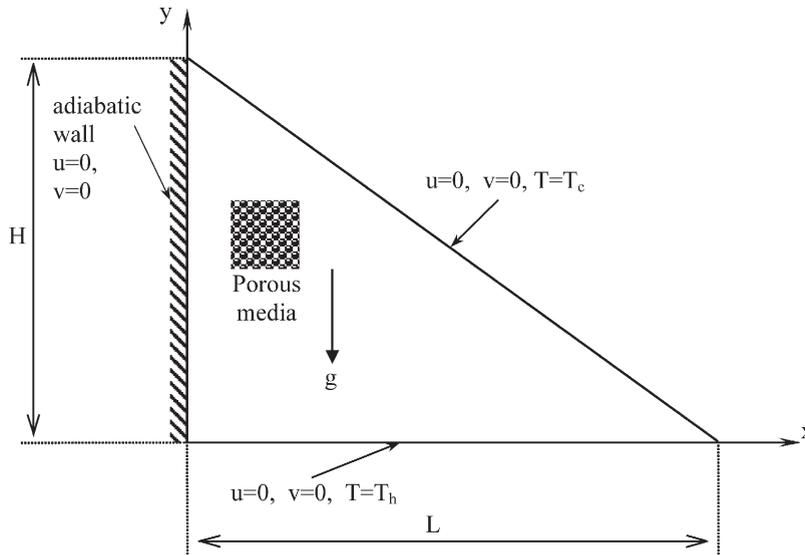


Fig. 1. Problem definition and coordinate system.

terms of the momentum equations are negligible [18]. With these assumptions, dimensional governing equations as continuity, momentum and energy can be written as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{g\beta K}{\nu} \frac{\partial T}{\partial x} \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{3}$$

The above equations are written in terms of the stream function defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{4}$$

and Eqs. (1)–(3) can be written in non-dimensional form as follow

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -Ra \frac{\partial \theta}{\partial X} \tag{5}$$

$$\frac{\partial \Psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \tag{6}$$

The non-dimensional parameters are listed as

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \Psi = \frac{\psi}{\alpha}, \quad Ra = \frac{(g\beta K (T_h - T_c)L)}{\nu \alpha} \tag{7}$$

Boundary conditions for the considered model are depicted on the physical model (Fig. 1). In this model,  $u=v=0$  for all solid boundaries.

$$\text{On the adiabatic wall, } 0 < y < H, \frac{\partial T}{\partial x} = 0 \tag{8}$$

$$\text{On the bottom wall, } 0 < x < L, T = T_h \tag{9}$$

$$\text{On the inclined wall, } T = T_C \tag{10}$$

Local and mean Nusselt numbers are calculated via Eq. (11)a and b, respectively.

$$Nu_x = \left( -\frac{\partial \theta}{\partial Y} \right)_{Y=0}, \quad Nu = \int_0^L Nu_x dx \tag{11a,b}$$

### 3. Numerical method

Finite difference method is used to solve governing equations (Eqs. (5) and (6)). Central difference method is applied for discretization of equations. The solution of linear algebraic equations was performed using Successive Under Relaxation (SUR) method. As convergence criteria,  $10^{-4}$  is chosen for all depended variables and value of 0.1 is taken for under-relaxation parameter. The number of grid points is taken as  $61 \times 61$  with uniform spaced mesh in both  $X$ - and  $Y$ -directions. The numerical algorithm used in this study was tested with the classical natural convection heat transfer problem in a differentially heated square porous enclosure. The obtained numerical results are compared with those given by different authors as tabulated in Table 1. As can be seen from the table, the obtained result shows good agreement with the results of literature. Contours of streamline and isotherms are almost the same to those given literature in rectangular enclosure but they are not presented here to save space.

### 4. Results and discussion

A numerical study has been performed in a right-angle triangular porous enclosure which is heated from the bottom and cooled from the inclined wall while vertical wall is insulated. Prandtl number is taken as  $Pr=0.71$  and governing parameters are Rayleigh number ( $50 \leq Ra \leq 1000$ ) and aspect ratio ( $0.25 \leq AR=H/L \leq 1.0$ ).

Streamlines and isotherms are presented for different Rayleigh numbers at  $AR=0.5$  in Fig. 2. The numbers of cells are increased with the increasing of Rayleigh number due to increasing of convection effects. In small Rayleigh numbers,  $Ra=50$  and  $100$ , single cell is formed in clockwise direction as indicated in Fig. 2a and b (on the left). On the contrary, multiple cells are obtained at high Rayleigh numbers,  $Ra=500$  and  $1000$ . The inclined boundary also plays important role on flow field because hot air moves from bottom to top and impinges to the inclined wall and distorted to different directions. Thus, stronger convection causes the increase on number of cells. Isotherms are moved to almost parallel to each other in low Rayleigh number due to domination of conduction regime which are presented in Fig. 2a and b. However, as convection heat transfer becomes stronger, the plumelike distribution is formed as seen in Fig. 2c and d.

Aspect ratio is important parameter which is defined the ratio of the height ( $H$ ) of vertical wall to the length of the bottom wall ( $L$ ) on flow field and temperature distribution. The effects of aspect ratio are presented by streamlines and isotherms for different aspect ratios at fixed Rayleigh number ( $Ra=1000$ ) in Fig. 3. Because of the high Rayleigh number, multiple cells are

Table 1  
Comparison of mean Nusselt number with literature at  $Ra=1000$

Paper	$Nu$
Bejan [12]	15.800
Cross et al. [14]	13.448
Manole and Lage [15]	13.637
Goyeau et al. [13]	13.470
Baytas and Pop [19]	14.060
Saeid and Pop [16]	13.726
This study	13.564

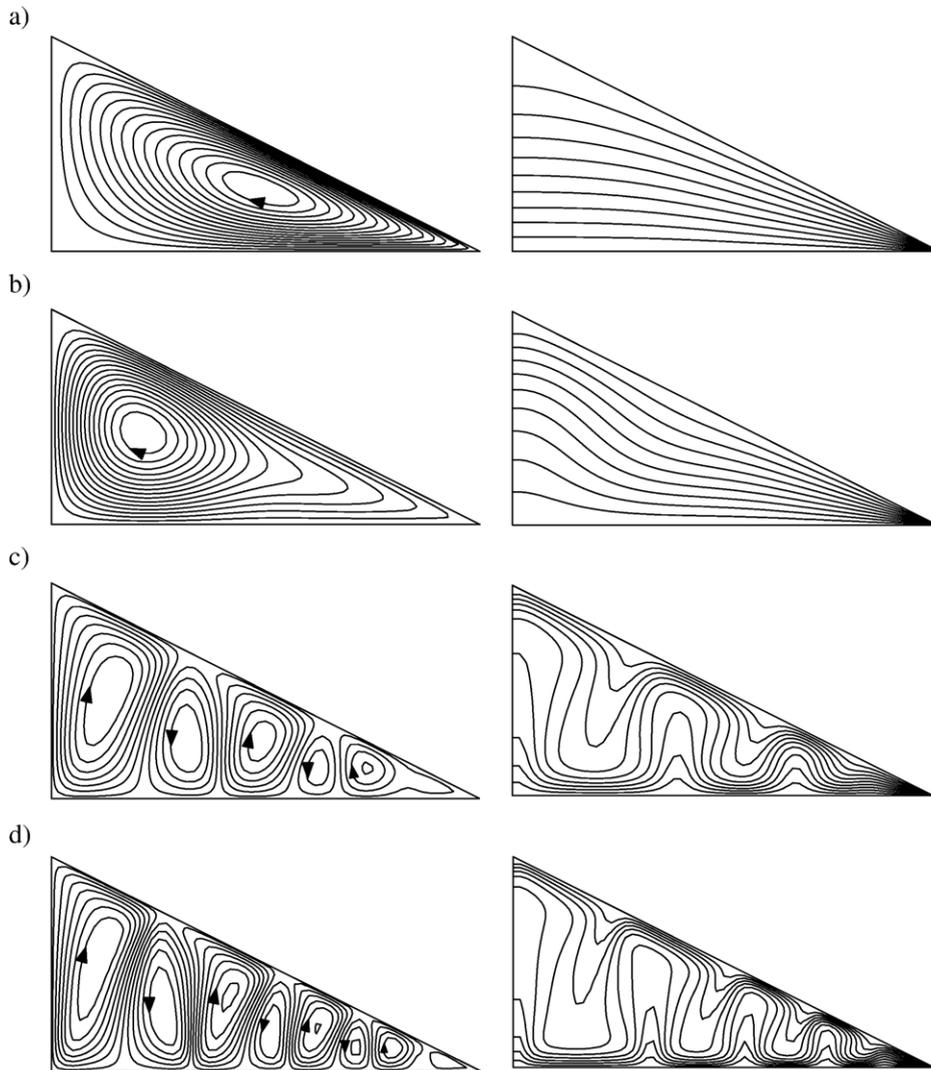


Fig. 2. Streamlines (on the left) and isotherms (on the right) for different Rayleigh numbers at  $AR=0.5$ , a)  $Ra=50$ , b)  $Ra=100$ , c)  $Ra=500$ , d)  $Ra=1000$ .

formed inside the triangular enclosure for all value of the aspect ratio as seen in streamlines (on the left column). However, the number of cells is decreased with the increasing of aspect ratio since the volume of triangular enclosure is increased with the increasing of aspect ratio. Vortexes are pushed each other due to geometrical shape of the enclosure. At the highest value of Rayleigh number the main cell is dominant to other cells. Isotherms (on the left column) show plumelike distribution in all value of the aspect ratio but the number of plume shapes is decreased with the increasing of aspect ratio (Fig. 3). It means that the heat transfer per unit length is decreased due to increasing distance between differentially heated walls. The variation of the local Nusselt number along the bottom wall is presented in Fig. 4a and b for different aspect ratios and Rayleigh numbers, respectively. Local Nusselt number show wavy variation at all value of aspect ratio and it is very high near the intersection of hot and cold wall. Both in left and right corners of the triangular enclosure, the value of local Nusselt number is higher at  $AR=0.25$  than that of other value of aspect ratio due to smaller distance between differentially heated walls. The highest peak value is obtained at  $x=0.65$  for  $AR=0.25$ . It is observed that peak values are high at the smallest aspect ratio. It is interesting that local Nusselt number values are higher at  $AR=0.75$  than that or  $AR=1$ . It means that there should be an optimum aspect ratio value which is effective on heat transfer. To obtain the effect of Rayleigh number on local Nusselt number is given at  $AR=0.5$  in Fig. 4b. As seen from the figure, the variation of local Nusselt number is increased linearly up to  $x=0.9$  at  $Ra=100$  due to conduction dominated regime. For the value of  $Ra>100$ , the wavy variation is formed on the variation of

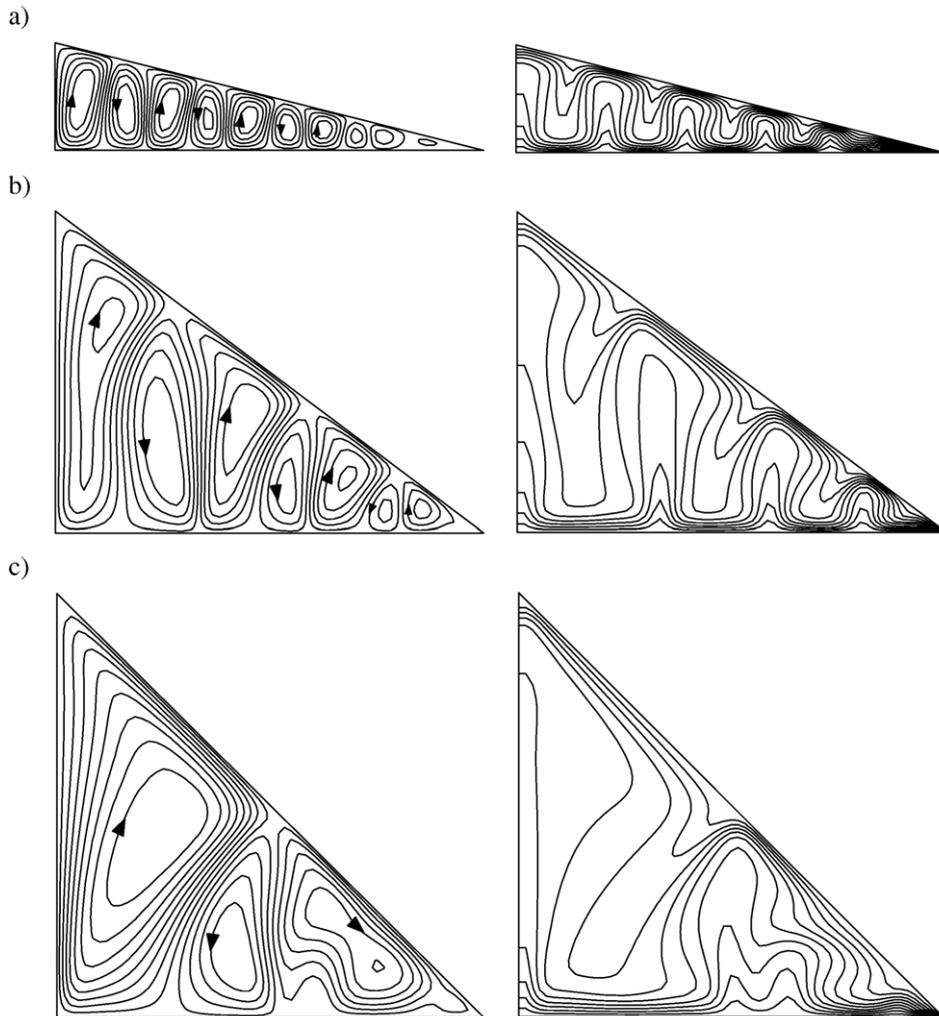


Fig. 3. Streamlines (on the left) and isotherms (on the right) for different aspect ratios at  $Ra=1000$ , a)  $AR=0.25$ , b)  $AR=0.75$ , c)  $AR=1$ .

local Nusselt number since convection effects become dominant. At  $x=0$  and  $x=0.97$ , the values are almost the same due to constant aspect ratio. Local Nusselt number has a peak value where the flow velocity is high. As indicated in the figure that peak values are increased with the increasing of  $Ra$  number due to high velocity as expected.

The overall heat transfer is presented via variation of mean Nusselt number and Rayleigh number in Fig. 5 for different aspect ratios. As indicated in the figure, the mean Nusselt number is increased linearly with the increasing Rayleigh number which is expected. The value of mean Nusselt number becomes smaller with the increasing of aspect ratio because, in the case of higher aspect ratio, the distance between differentially heated wall is increased. The highest mean Nusselt number value is obtained at the highest Rayleigh number and the lowest value of aspect ratio.

## 5. Conclusions

In this study, natural convection heat transfer was analyzed in a right-angle triangular enclosure. Natural convection forms when bottom wall is heated and the inclined wall of triangle is cooled while vertical wall is adiabatic. Heat transfer is increased with the increasing of Rayleigh number. It is observed that multiple cells are formed in a right-angle triangular enclosure at high Rayleigh number due to convection domination regime. In addition, single cell is obtained in low Rayleigh number since conduction heat transfer regime becomes dominant.

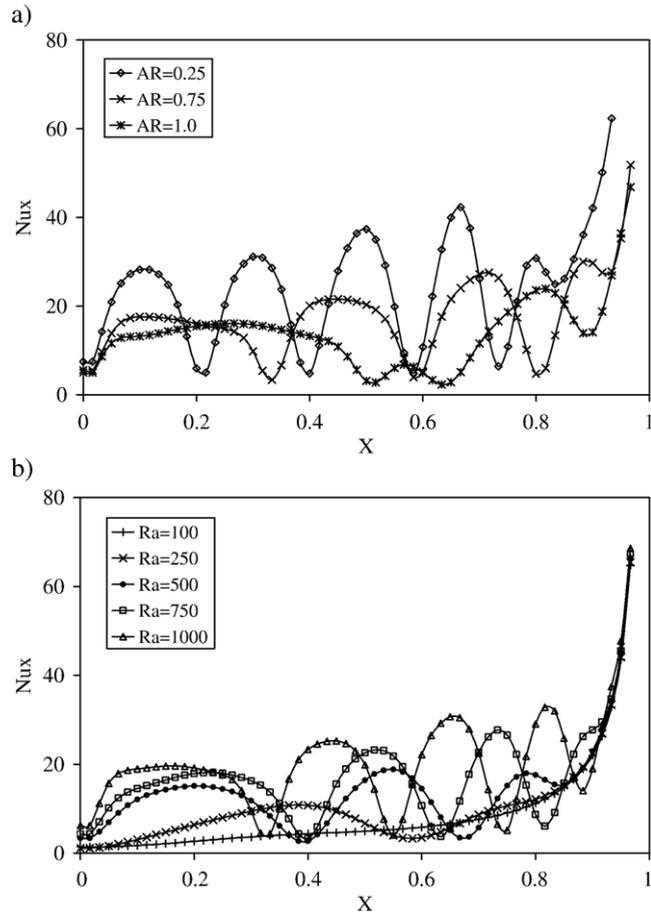


Fig. 4. Variation of local Nusselt number along the hot wall, a)  $Ra=1000$ , for different aspect ratios, b)  $AR=0.5$ , for different Rayleigh numbers.

The local Nusselt number shows wavy variation along the bottom wall and it has maximum value at the right corner of triangle where temperature difference is high. The maximum mean Nusselt number is obtained for the highest Rayleigh number and the lowest aspect ratio.

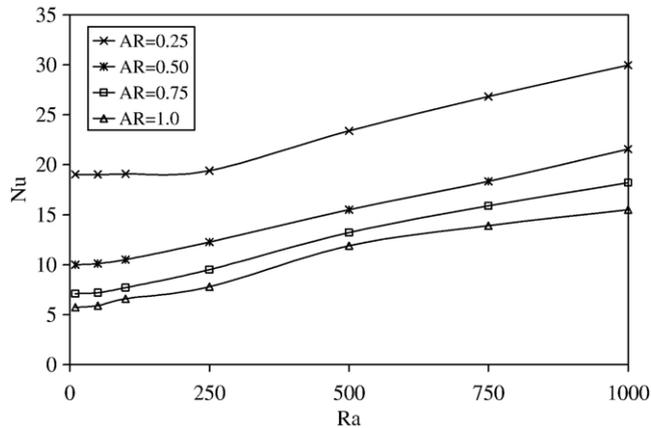


Fig. 5. Variation of mean Nusselt number with different Rayleigh numbers for  $AR=0.25, 0.50, 0.75$  and  $1.0$ .

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