POLS 5377 Scope & Method of Political Science

Week 6 Normal Curve

Normal Curve

Healey. (2016) *Statistics: A Tool for Social Research,* Chapter 5

Key Question:

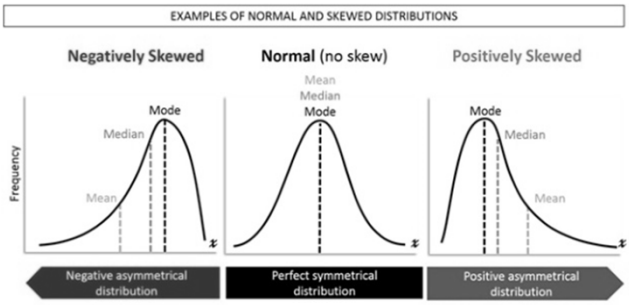
- What is the concept of normal curve?
- How to compute Z scores and use the normal curve table to find the area with the given points
- How to estimate the probability that certain scores will full under the normal distribution?

Outline

- Properties of the Normal Curve
- Computing Z Scores
- Using the Normal Curve Table
- Estimating Probabilities

Properties of Normal Curve

- When present a variable frequency distribution in diagram, we can see the distribution curve. There are all types shape of curve may be observed.
- Statisticians found that normal curves are commonly observed in the real world.

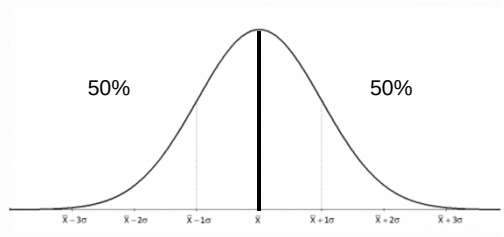


Properties of Normal Curve

- The statisticians are very interested in the normal curve, and even standardized it for comparing frequency distributions across variables.
- Normal curve, Normal distribution, standardized normal distribution all refer to the same thing.
- Why standardize the normal curve?
  - To compare among different distributions.
  - Consider the example of percentage and currency

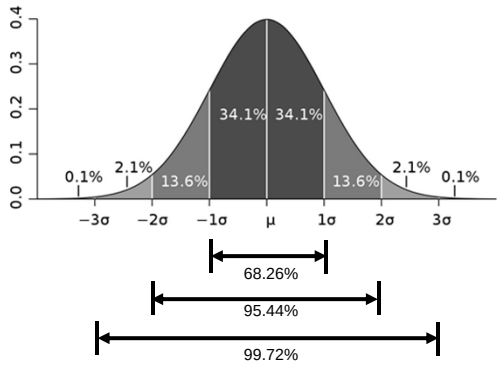
Properties of Normal Curve

- Theoretical
- Bell-shaped
- Smooth curve
- Symmetrical
- Unskewed
- Tails extend to infinity
- Mode, median, and mean are same value



Properties of Normal Curve

- Most important property: distances on horizontal axis always cut off the same area, when measured by the **standard deviation**
  - ±1 σ = 68.26%
  - ±2 σ = 95.44%
  - ±3 σ = 99.72%
  - +1 σ = 34.14%
  - +2 σ = 47.72%
  - +3 σ = 49.86%

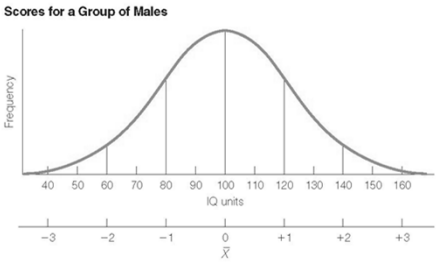


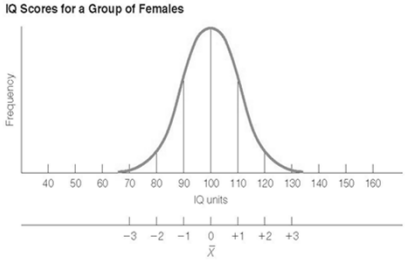
Properties of Normal Curve

- Examples: IQ test results for male and female, both are normally distributed, but with different standard deviations.

Males	Females
$\overline{X}$ = 100	$\overline{X}$ = 100
s = 20	s = 10
N = 1000	N = 1000

S = standard deviation





Computing Z Scores

- Z scores are scores that have been standardized to the theoretical normal curve
- Z scores represent how different a raw score is from the mean in standard deviation units
- To find areas, first compute Z scores
- The Z score formula changes a raw score to a standardized score

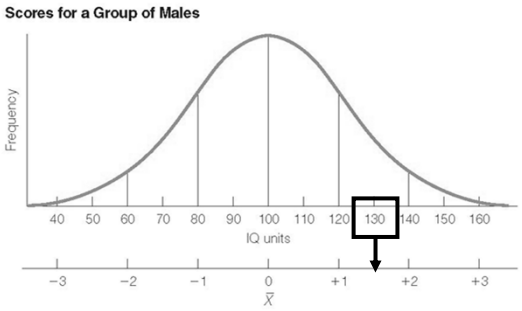
$$z = \frac{X_i - \overline{X}}{s}$$

Computing Z Scores

- An IQ score of 130 falls one standard deviation above the mean

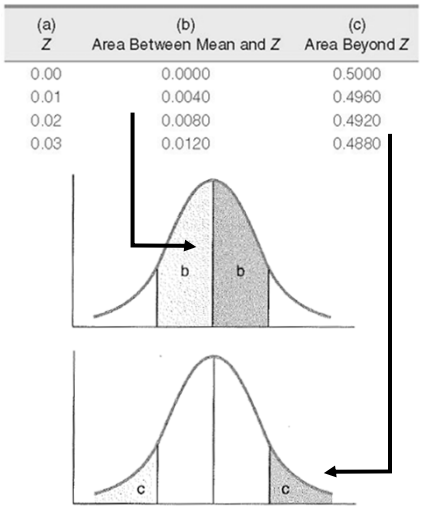
$$Z = \frac{X_i - \bar{X}}{S} = \frac{130 - 100}{20} = 1.5$$

Males	Females
$\overline{X}$ = 100	$\overline{X}$ = 100
s = 20	s = 10
N = 1000	N = 1000



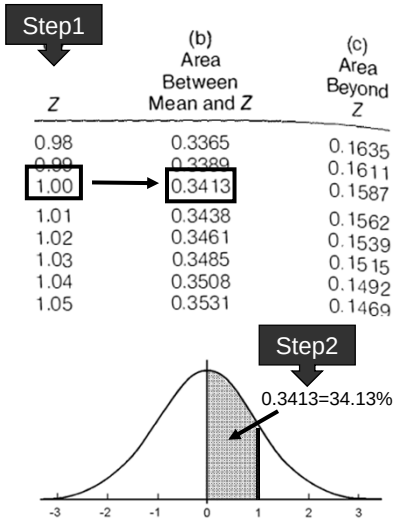
Using the Normal Curve Table

- Appendix A: The table for finding the area under the normal curve (pg.443, 10th ed., or pg. 469, 9th ed.)
- The table has three columns
  - Column A = Z scores
  - Column B = areas between the Z score and the mean
  - Column C = areas beyond the score Z into the tail



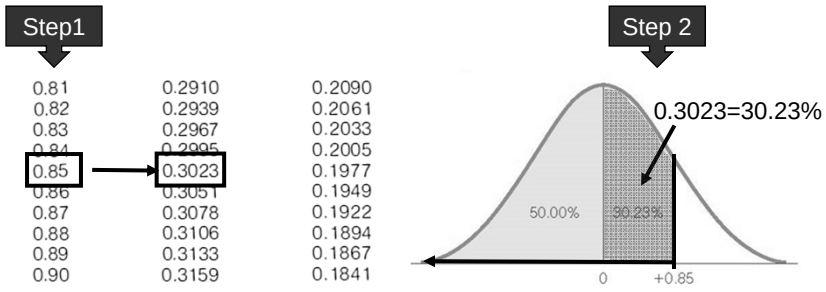
Using the Normal Curve Table

- Appendix A can be used to find the areas above and below a score or between scores
  - First compute the Z score, taking careful note of the sign of the score.
  - Draw a picture of the normal curve and shade in the area in which you are interested.



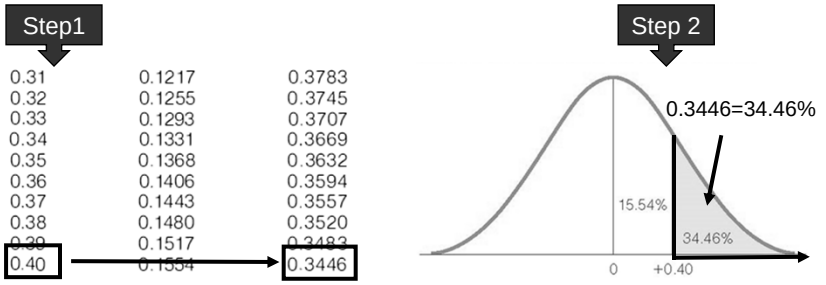
Using the Normal Curve Table

- Example: find the area *below* a positive Z score: 0.85
  - Step 1: Column (a): z=0.85, Column (b): 0.3023
  - Step 2: Mark the area between the mean and z=0.85, and plus the area below the mean, 0.5 or 50%.
  - Result: 0.5+0.3023=0.8023 or 30.23%+50%=80.23%



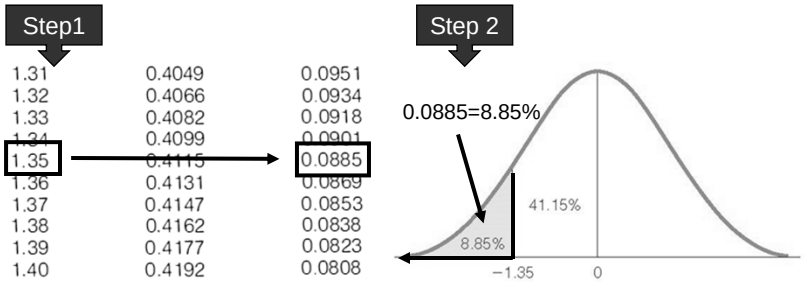
Using the Normal Curve Table

- Example: find the area *above* a positive Z score: 0.40
  - Step 1: Column (a): z=0.40, Column (c): 0.3446
  - Step 2: Mark the area beyond the z=0.40.
  - Result: Area above z (= 0.40) is 0.3446 or 34.46%



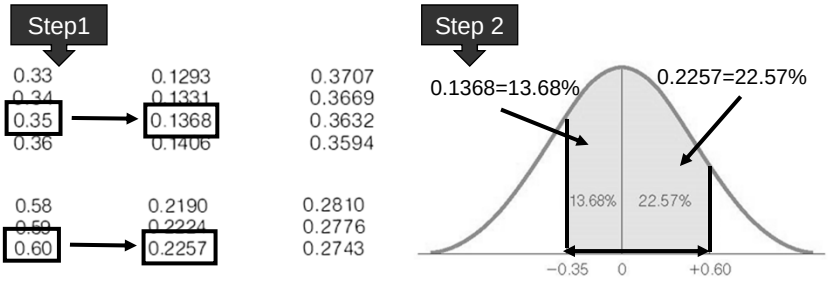
Using the Normal Curve Table

- Example: find the area *below* a negative Z score: -1.35
  - Step 1: Column (a): z=1.35, Column (c): 0.0885
  - Step 2: Mark the area below the z=-1.35.
  - Result: Area above z (= -1.35) is 0.0885 or 8.85%



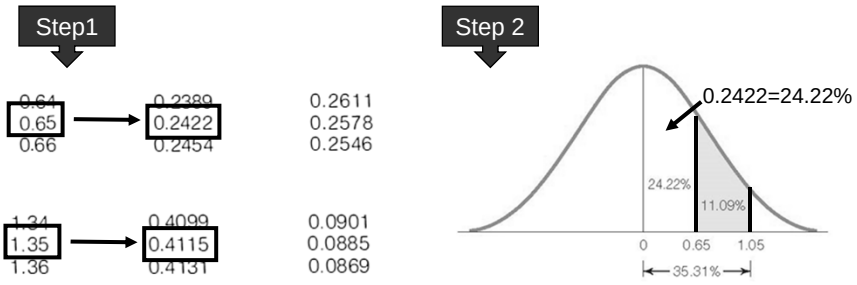
Using the Normal Curve Table

- Example: find the area *between* the z score of -0.35 and 0.60
  - Step 1: Column (a): z=0.35 & 0.60, Column (b): 0.1368 & 0.2257
  - Step 2: Mark the area between the z points.
  - Result: Area between z -0.35 and 0.60 is: 0.1368+0.2257=0.3625 or 36.25%



Using the Normal Curve Table

- Example: find the area *between* the z score of 0.65 and 1.05
  - Step 1: Column (a): z=0.65 & 1.05, Column (b): 0.2422 & 0.3531
  - Step 2: Mark the area between the z points.
  - Result: Area between z 0.65 and 1.05 is: 0.3531- 0.2422=0.1109 or 11.09%



Estimating Probabilities

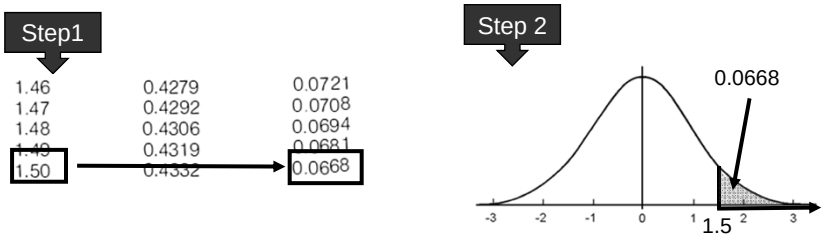
- Areas under the curve can also be expressed as probabilities
- Probabilities are proportions and range from 0.00 to 1.00
- The higher the value, the greater the probability (the more likely the event)
- The chances that a success could theoretically occur

$$\text{Probability} = \frac{\#\ successes}{\#\ events}$$

  - Numerator - the number of events that would constitute a success
  - the denominator - the total number of possible events
- When flip a coin once, the probability of getting a head is $\frac{1}{2} = 0.5$.
- When draw a card from a deck of cards. The probability of drawing an ACE from 52 cards, is $\frac{4}{52} = 0.0769$. (There are 4 aces in a deck of cards.)

Estimating Probabilities

- Mean = 13; s = 4. The probability of selecting a score of *19 or more*:
  - Step 1: find the z score $Z = \frac{X_i - \bar{X}}{S} = \frac{19-13}{4} = \frac{6}{4} = 1.5$
  - Step 2: use the normal curve table to find z score 1.5 and the area beyond (column c): 0.0668
  - Result: probability of selecting a score of *19 or more* is 0.0668.



After this lecture:

You should be able to:

- Describe and explain the concept of the normal curve
- Covert empirical scores to Z scores and use Z scores and the normal curve table to find areas below, and between points on the curve
- Express areas under the curve in terms of probabilities