POLS 5377 Scope & Method of Political Science

Week 13 Hypothesis Testing IV

# Chi Square

Healey. (2016) *Statistics: A Tool for Social Research,* Chapter 11

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# Key Questions:

- What is a cross-tabulation table?
- What is the logic of chi square?
- How to conduct a chi square hypothesis test, and interpret the results?
- How to conduct chi square tests and interpret the results within SPSS?
- What are the limitations of using chi square tests?

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# Outline

- Bivariate (Cross tabulation) Tables
- Logic of Chi Square
- The Computation of Chi Square
- The Chi Square Test within SPSS
- Limitation of the Chi Square Test

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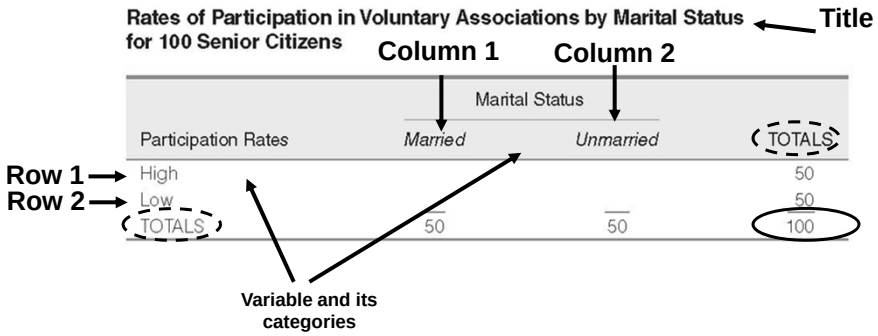
# Bivariate Table

- Bivariate table = Cross tabulation table = Contingency table
  - display the scores of cases on two different variables at the same time
  - A researcher is studying membership in voluntary associations and hypothesizes that unmarried people will be more involved because they have fewer family obligations and more free time.



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# Bivariate Table

| Title | | | |
|---|---|---|---|
| Rows (**Dependent Variable**) | Columns (**Independent Variable**) | | Total |
| Row 1 | cell a | cell b | Row **Marginal 1** |
| Row 2 | cell c | cell d | Row **Marginal 2** |
| Total | Column **Marginal 1** | Column **Marginal 2** | N |

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# Bivariate Table

- There are two dimensions: rows and columns
  - Columns are scores of the *independent* variable
    - There will be as many columns as there are categories on the independent variable
  - Rows are scores on the *dependent* variable
    - There will be as many rows as there are categories on the dependent variable
  - There will be as many cells as there are categories on the two variables combined
  - Each cell reports the number of times each combination of categories occurred

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# Logic of Chi Square ($\chi^2$)

- Chi Square as a test of statistical significance is a test for **independence**
  - Independence: *Two* variables are independent if the classification of a case into a particular category of one variable has no effect on the probability that the case will fall into any particular category of the second variable" (p. 274 in 9th edition; p. 279 in 10th edition)
- The chi square test is a procedure for evaluating the level of statistical significance of a relationship between *two variables* in a bivariate table or contingency table (also called crosstabs).
- The chi square procedure assumes that there is **no relationship** between two variables in the population (null hypothesis).

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# Logic of Chi Square ($\chi^2$)

- We are looking for significant differences between the actual cell frequencies observed in a table ($f_o$) and those that would be expected by random chance or if cell frequencies were independent ($f_e$)

$$\chi^2(obtained) = \sum \frac{(f_o - f_e)^2}{f_e}$$

- $f_o$: the cell frequencies observed in the bivariate table
- $f_e$ : the cell frequencies that would be expected if the variables were independent

$$f_e = \frac{\text{Row marginal} \times \text{Column marginal}}{N}$$

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# Logic of Chi Square ($\chi^2$)

- Example
  - Is there a relationship between the **accreditation status** of social work programs and the **employment status** of their students?
  - 100 students were selected and asked 1) if they graduate from a accredited / non-accredited program? 2) if they are working as a social worker?

**Employment of 100 Social Work Majors by Accreditation Status of Undergraduate Program**

| Dependent variable ↓ / Independent variable → Employment Status | Accreditation Status: *Accredited* (Observed Frequencies) | *Not Accredited* | TOTALS |
|---|---|---|---|
| Working as a social worker | 30 | 10 | 40 |
| Not working as a social worker | 25 | 35 | 60 |
| TOTALS | 55 | 45 | 100 |

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# Logic of Chi Square ($\chi^2$)

- Expected frequency ($f_e$) for the top-left cell

$$f_e = \frac{\text{Row marginal} \times \text{Column marginal}}{N} = \frac{40 \times 55}{100} = 22$$

**Expected Frequencies for Table 11.3**

| Employment Status | Accreditation Status: *Accredited* (Expected Frequency) | *Not Accredited* | TOTALS |
|---|---|---|---|
| Working as a social worker | 22 | 18 | 40 |
| Not working as a social worker | 33 | 27 | 60 |
| TOTALS | 55 | 45 | 100 |

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# Logic of Chi Square ($\chi^2$)

- Compute obtained $\chi^2$

$$\chi^2(obtained) = \sum \frac{(f_o - f_e)^2}{f_e}$$

**Computational Table for Table 11.3**

| (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|
| $f_o$ | $f_e$ | $f_o - f_e$ | $(f_o - f_e)^2$ | $(f_o - f_e)^2/f_e$ |
| 30 | 22 | 8 | 64 | 2.91 |
| 10 | 18 | −8 | 64 | 3.56 |
| 25 | 33 | −8 | 64 | 1.94 |
| 35 | 27 | 8 | 64 | 2.37 |
| 100 | 100 | 0 | | 10.78 |

**Obtained $\chi^2$**

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# Five-Step Model - Chi Square

**Step 1: Make Assumptions and Meet Test Requirements**

- Independent random samples
- Level of Measurement is nominal
  - Accredited program / Non-accredited program
  - Working as a social worker / not working as a social worker

(Note: In chi square hypothesis test, we don't need to assume the normal distribution of the population or sampling distribution)

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# Five-Step Model - Chi Square

**Step 2: State the Null Hypothesis & Alternative Hypothesis**

- $H_o$: $f_o = f_e$ The accreditation status of program and the student employment status are **independent**
  - Null hypothesis always asserts the variables are independent
- $H_1$: $f_o \neq f_e$ The accreditation status of program and the student employment status are **dependent**
  - Alternative hypothesis always contradicts to null hypothesis, and asserts the variables are dependent

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# Five-Step Model - Chi Square

**Step 3: Select the Sampling Distribution and Establish the Critical Region**

- Sampling Distribution = Chi Square distribution (Appendix C)
- Alpha = 0.05
- Degree of freedom (df) = (Number of rows - 1)* (Number of colums-1) = (2-1)*(2-1)=1
- $\chi^2$(critical) = 3.841
  - The score marks the beginning of the critical region

| df | 0.99 | … | 0.05 | … |
|---|---|---|---|---|
| 1 | … | … | **3.841** | … |
| 2 | … | … | 5.991 | … |
| 3 | … | … | 7.815 | … |

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# Five-Step Model - Chi Square

**Step 4: Calculate the Test Statistic**

- Calculate $\chi^2$(obtained) with the formula:

$$\chi^2(obtained) = \sum \frac{(f_o - f_e)^2}{f_e}$$

- In the example of program accreditation status and student employment status, $\chi^2$(obtained) = 10.78

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# Five-Step Model - Chi Square

**Step 5: Make a Decision and Interpret the Results of the Test**

- $\chi^2$(critical) = 3.841
- $\chi^2$(obtained) = 10.78
- The obtained $\chi^2$ score falls in the critical region, so **reject** $H_0$
  - There is a significant relationship between the two variables.
- Report results:
  - At the significance level of 0.05, the obtained $\chi^2$ score falls in the critical region, so we **reject** $H_0$. The data suggests there is a significant relationship between employment status and accreditation status in the population from which the sample was drawn.

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# Interpreting Chi Square

- Need to interpret chi square test results with the following understanding:
  - The chi square test tells us ONLY if the variables are independent or not
  - It does not tell us the pattern or nature of the relationship
  - To investigate the pattern, we need to compute the percentages within each column and compare across the columns

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# Interpreting Chi Square

- Column percentage make the relationship between the two variables more obvious:
  - The students from accredited programs are more likely to be working as social workers.
  - 55% of the students from accredited programs working as social works versus only 22% of the students from non-accredited programs working as social workers
  - According to the test results, we know this relationship is significant. It doesn't occur by chance.

| Employment status | Accreditation Status | | |
|---|---|---|---|
| | Accredited | Not accredited | |
| Working as a social worker | 30 (54.55%) → **(30/55)*100%** | 10 (22.22%) → **(10/45)*100%** | 40 |
| Not working as a social worker | 25 (45.55%) → **(25/55)*100%** | 35 (77.78%) → **(35/45)*100%** | 60 |
| | 55 (100%) | 45 (100%) | 100 |

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# Chi Square with SPSS

- In practice, we usually don't calculated a chi square by hand, and we rely on statistical software such as SPSS.
- Now I will show you how to conduct a chi square test with SPSS and interpret the results
- We want to examine the relationships between the **education level** and individuals' **tolerance toward Muslim clergy** in U.S. society.
- Conduct "Crosstabs" procedure in SPSS (pg. 301@10th ed. OR pg. 298 @ 9th ed.)
  - Dataset: GSS2012_Student_B
  - [Analyze] → [Descriptive Statistics] → [Crosstabs]
  - Place dependent variables in the **Row** box
  - Place independent variables in the **Column** box
  - Click the **Statistics** button, select **Chi-square**
  - Click the **Cells** button, select **column** in the **Percentage** box
  - Proceed the analysis

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# Chi Square with SPSS

- Dependent variable: Tolerance toward Muslim clergy (muslim_tol)
- Independent variable: Education (educ_2)

Step 1: Meet requirements

- Independent random sample
- Level of measurement: nominal

Step 2: State Null hypothesis and alternative hypothesis

- $H_o$: $f_o = f_e$ The tolerance toward Muslim clergy and the education level are **independent**
- $H_1$: $f_o \neq f_e$ The tolerance toward Muslim clergy and the education level are **dependent**

Step 3: Critical Region

- $\alpha$=0.05

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# Chi Square with SPSS

**Tolerance twrd Muslim clergy * Education: 2 Cats Crosstabulation**

| | | | Education: 2 Cats | | |
|---|---|---|---|---|---|
| | | | 0-12 yrs | 13+ yrs | Total |
| Tolerance twrd Muslim clergy | Low | Count | 230 | 173 | 403 |
| | | % within Education: 2 Cats | 57.2% | 32.3% | 43.0% |
| | Middle | Count | 97 | 173 | 270 |
| | | % within Education: 2 Cats | 24.1% | 32.3% | 28.8% |
| | High | Count | 75 | 190 | 265 |
| | | % within Education: 2 Cats | 18.7% | 35.4% | 28.3% |
| Total | | Count | 402 | 536 | 938 |
| | | % within Education: 2 Cats | 100.0% | 100.0% | 100.0% |

- Degree of freedom (df) = (R-1)*(C-1)=(3-1)*(2-1)=2
- $\chi^2$(obtained) = 61.472
- Test result: significance value = .000 < $\alpha$=0.05
  - Reject $H_0$
  - There is a significant relationship between the two variables.

**Chi-Square Tests**

| | Value | df | Asymp. Sig. (2-sided) |
|---|---|---|---|
| Pearson Chi-Square | 61.472[a] | 2 | .000 |
| Likelihood Ratio | 62.169 | 2 | .000 |
| Linear-by-Linear Association | 57.867 | 1 | .000 |
| N of Valid Cases | 938 | | |

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 113.57.

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# Chi Square with SPSS

- Interpret the result:
  - At the significance level of 0.05, the test of chi square is significant, so we **reject** $H_0$. The data suggests there is a significant relationship between tolerance toward Muslim clergy and education level in the U.S society.
  - According to the column percentages, the citizens with lower education level are more likely to have a low tolerance toward Muslim clergy.
  - 57.2% of the individuals who received 0 to 12 years education have low tolerance toward Muslim clergy, versus 32.3% of those received 13 years or more education.

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# Limitation of Chi Square

- Difficult to interpret when variables have many categories
  - Best when variables have four or fewer categories
- With small sample size, cannot assume that Chi Square sampling distribution will be accurate
  - Small sample: High percentage of cells have expected frequencies of 5 or less
- Like all tests of hypotheses, Chi Square is sensitive to sample size
  - As N increases, obtained Chi Square increases
  - With large samples, trivial relationships may be significant

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# After this lecture:

You should learn the following key concepts:

- How to construct and interpret a crosstab table
- The logic of chi square test
- The condition in which conducting a chi square test is appropriate
- How to compute and test chi square manually and with SPSS
- How to interpret the chi square test results
- Limitation of chi square test